CHAPTER 10

Nicomachus

Nicomachus belongs to roughly the same period as Adrastus and Theon. His exact dates are not known, but he was probably active around the beginning of the second century A.D. About his life the only clear fact we have is that he was born in Gerasa (even that is more problematic than it might seem, since there were several cities with that name). He drops a few autobiographical hints in the essay translated here, but they add up to very little. The work for which he is best known is his *Introduction to Arithmetic*. Though modern commentators agree that it shows him as no more than a second-rate mathematician, the book is a useful compendium of Greek studies in number theory. Through a Latin translation by Apuleius in the second century A.D., it became the Romans' principal source of knowledge about Greek achievements in this field. Among Nicomachus' other writings were an introduction to geometry and a biography of Pythagoras; both are lost. He also wrote a treatise called *Theologoumena Arithmeticae* ('Theology of Number') of which we have only a cursory summary by Photius, but an anonymous work with the same title survives, much of which is certainly derived from Nicomachus. His essay was evidently an elaborate presentation of Pythagorean theories about the symbolic and mystical significance of numbers, such as are set out much more briefly in 12 Arist. Quint. *De Mus*. Book III ch. 6.

The *Enchiridion* or 'handbook' of harmonics has the honour of being the only work on the subject to have survived complete from the period between Euclid and Ptolemy. Like the *Introduction to Arithmetic*, it is not an intellectually distinguished or original piece, but it had an important influence on later writers (notably Boethius and Bryennius, who quote it extensively). It is, by its own statement, a mere 'introduction' to harmonics, and promises a fuller treatment at a later time. We do not know whether the larger work was completed: some extracts that may be drawn from it exist in a collection of MSS pages printed by Jan in *MSG* under the title *Excerpta ex Nicomacho*.

The programme of the *Enchiridion* differs markedly from that of any Aristoxenian harmonic treatise, and has little in common with our earliest intact Pythagorising specimen, the *Sectio Canonis*. By the standards of either, it is eccentric and incomplete, and though it includes what are now valuable if obscure accounts of some non-standard scalar systems, it makes no significant original contributions to harmonic theory. For these and other reasons one modern scholar (Levin 1975) has concluded that it is not properly considered as a treatise in harmonics at all, but rather as a piece of propaganda for the Pythagorean world-view in general. She takes Nicomachus' presentation of it in the guise of an elementary manual of harmonics to be little more than a pretext.

This interpretation has something to be said for it, but I think it is exaggerated. The work has a clear pattern of development, different from those mentioned above, but perfectly appropriate to an essay of the kind it purports to be. It is true that the material is given a thoroughly Pythagorean slant, though not one that is rigorously mathematical in Euclid's style. It appears not only in the author's insistence on basing harmonics in physical acoustics, in his use of the language of ratios, and in his emphatic references to Pythagoras himself, but also in his attempts to link the foundations of musical 'harmony' to those underlying the workings of the universe at large. Such an approach is to be expected from the writer of the *Theologoumena Arithmeticae*. Nicomachus'

reverence towards number and his belief in its role as a divine principle are evident, even though he makes no use here of the numerological symbolism found in the 'theological' treatise. But like the writings of Adrastus (9.2–9.3), the *Enchiridion* draws heavily (without acknowledgement) on Aristoxenian forms of analysis too, 'correcting' them in certain instances, and unrolls quite systematically as an exposition of the simplest and most basic structures that Aristoxenus himself identifies, re-expressed and explicated in Pythagorean terms. There are admittedly many concepts and structures of which it says nothing. These, it can fairly be assumed, are reserved for the fuller treatment which it promises, and some of them are hinted at in the final chapter. The topics it does pursue do not differ much in scope from those addressed by Adrastus.

A brief summary of the subjects considered will give the gist of its design. After a short introduction (chapter 1), Nicomachus first discusses the distinction between continuous and intervallic vocal movement, and the kinds of 'space' within which each occurs. His material is drawn from Aristoxenus (7 *El. Harm.* 8.13ff.), with modifications that import little but confusion (chapter 2). Chapter 3 introduces the thesis that music as we know it is derived from that of the heavens, and sketches a planetary scale of seven notes, consisting of two tetrachords in conjunction. Chapter 4 gives an account of the causes of sound and pitch, designed to justify the doctrine (already touched on in the celestial context in chapter 3) that notes differ and are related to one another 'in accordance with number', adducing in evidence the acoustic properties of instruments, which it distinguishes into three groups.

These early chapters have set the scene for the development of harmonic investigations in the Pythagorean style. In the next five, Nicomachus gradually introduces and confirms an analysis of the simplest and most important extended harmonic structure, the diatonic octave system. He begins by explaining (chapter 5) that the system to be analysed is not the seven-note scale of chapter 3, but an eight-note scale whose invention he attributes to Pythagoras, and he sketches its general shape, adding a reference to the ratios fundamental to its construction (those of the concords and the tone). Chapter 6 tells the story of Pythagoras' discovery of these basic ratios, and speaks of their confirmation in experiments with various instruments. He has now prepared us for a full analysis of the diatonic octave (in the form familiar from Philolaus, Plato and the *Sectio Canonis*). This is given in chapter 7, together with a parenthetical reference to the other two genera (more fully treated in chapter 12). The seven-note and eight-note scales are again compared. This stage of Nicomachus' account is completed in chapters 8–9, in which the evidence of Plato (misquoted and misinterpreted) and of Philolaus is called on to confirm the analysis. (Nicomachus therefore finds it necessary to add an explication of the system of Philolaus, a seven-note scale differing from those previously mentioned.) The two chapters can be construed as an attempt to underwrite the credentials of the 'Pythagorean diatonic octachord', first from the point of view of mathematics and metaphysics, and secondly from the facts of musical history, as presented in the earliest Pythagorean account that Nicomachus knew.

Chapter 10 returns to the properties of instruments, expanding what has been said in chapter 4, and discussing the relation of 'converse qualification' that holds between tensions and pitches on the one hand and lengths of string or pipe on the other (cf. 9.4 Thrasyllus ap. Theon Smyrn. 87). The chapter is best treated as a digression, but in view of the sequel is a relevant reminder of the conceptual apparatus involved in the Pythagorean approach.

The last two chapters pick up the programme of analysis where chapter 9 had left it. Chapter 11 sets out the form of the diatonic double octave, giving the names of the notes and tetrachords, the sizes of the intervals involved, and an account of the conjunctions and disjunctions between tetrachords. The terminology here is mainly Aristoxenian, but

among the further details that the last sentences of the chapter mention and postpone for a fuller treatment elsewhere, is a 'division of the *kanōn*' in proper Pythagorean style. The task of chapter 12 is to complete the elementary division of tonal space by an account of the progressions proper to the chromatic and enharmonic genera. But it begins with a brief account of some basic harmonic conceptions (note, interval, relation, difference, *systēma*, concord, discord). These remarks are said to be 'reminders', though in fact they correspond to nothing earlier in the text (but see n. 29 below), and apart from the distinction between 'difference' and 'relation' the sketches they offer are drawn from Aristoxenian models. Little use is made of them in the remainder of the chapter. Perhaps Nicomachus was merely uneasy at having so far said nothing to explicate these apparently important notions, and fitted them in here as best he could. The descriptions of the chromatic and enharmonic systems follow, again in thoroughly Aristoxenian terms (though only their simplest forms are considered, as in Adrastus and Thrasyllus). Nicomachus covers his tracks with a paragraph explaining, in effect, that when he spoke of 'semitones' and 'quarter-tones' in giving these descriptions he was not to be taken literally. He subscribes to the Pythagorean doctrine that tones cannot be divided into equal parts, though he does not explain why. The work ends with a list of the names of all the notes of the double octave system in all three genera, and a valedictory paragraph that once again promises a fuller treatment at a later date.

The details of Nicomachus' treatise are sometimes muddled, often naive. But its overall programme makes good sense as an elementary introduction to harmonics, interpreted to fit a particular form of Pythagoreanism. From a harmonic point of view, this 'Pythagoreanism' maintains a standard of metaphysical purity (or oversimplification) whose source is Plato, in that its principal systems are all direct derivatives of the 'cosmic scale' in 2.3 *Tim.* 35b–36b. No allowance is made for the existence of scales whose analysis involves more complex manoeuvres in the theory of ratio and proportion. Here the Platonising branch of the tradition differs sharply from that which leads from Archytas through Eratosthenes and Didymus to Ptolemy (see the Appendix to chapter 1). In this respect, as in its scope and method, the *Enchiridion* has clear affinities with the writings of Adrastus, Theon and perhaps Thrasyllus (see 9.1–9.5), and is perhaps scarcely more original in general conception than in its details. Its greater influence on musical writers of later antiquity is due less to its scientific merits than to Nicomachus' admirable talents as a populariser. Non-specialists, curious about the musical aspects of Pythagorean philosophy, could find in Nicomachus a straightforward and quite charming exposition of ideas which others had surrounded with mathematical complexities, or had hidden in technical commentaries on the dialogues of Plato.

On the life and works of Nicomachus see especially D'Ooge (1926). The most substantial study of the *Enchiridion* in English is Levin (1975).

10 The *Enchiridion*

Chapter 1 That this book is a manual serving as a memorandum of the elementary teachings of Harmonics

Though the description of the intervals and relationships involved in the elements of harmonics is in itself complex and difficult to pull together in a single memorandum, and though I especially, because of the restlessness and hurry of a traveller's life, am unable to devote myself with unruffled attention and mental concentration to the teaching of these matters, whose proper clarification demands above all both an opportune moment and careful

reasoning, with leisure and without distraction, nevertheless, best and noblest of women, I must arouse my greatest efforts, since it is you who have bidden
238 me at least to set out the major propositions for you in simple form, without elaboration or complex demonstrations, and to do so at once, so that by having these propositions collected in a single synopsis, and by using these brief notes as a manual, you may remind yourself of what is stated and taught there in rough outline under each heading.[1] If the gods are willing, as soon as I have some leisure and a break from my travels, I will put together for you a longer
10 and more accurate introduction to these matters, connected together with a fully reasoned argument, as the saying goes: it will be in several books, and I shall send it to you at the first opportunity, wherever I am told that you are living.[2] But now, to make my exposition easy to follow, I shall begin from the same place where I began my instruction when I was expounding these things to you in person.

Chapter 2 Concerning the two forms of vocal sound, the intervallic and the continuous, and the spaces occupied by each

Those belonging to the Pythagorean school used to say that there are two types
20 of human vocal sound, standing as species of a single genus.[3] Their technical word for the one was 'continuous', for the other 'intervallic', terms which they assigned to each on the basis of its special qualities. They took the intervallic to be that used in song, which comes to rest at each pitch, and makes plain the changes between each of its parts, which does not confusedly run together, and
239 which is articulated and divided by the magnitudes which go with each note, where the parts of the vocal sound lie beside one another as it were in an aggregation, not in a blended mixture, being readily separated and distinguished, and in no way melted into one another.[4] For sound in song is such as to display all the notes clearly to those who understand the subject, showing

[1] The identity of the lady to whom the treatise is addressed is unknown. The vast, sprawling sentence with which it opens is typical of Nicomachus. I have not always translated his sentences in a way that preserves their original shape and length, but have kept this one in roughly its original form as a specimen. For another extraordinary example see the beginning of chapter 5.

[2] On the promised major work see the introduction to this chapter. The opening address, giving the *Enchiridion* the form of a letter, tells us virtually all that is known of Nicomachus' life.

[3] The claim that this distinction is due to the Pythagoreans is disingenuous. The fundamental source for the whole passage is Aristoxenus, as its language plainly shows (see 7 Aristox. *El. Harm.* 3.5ff., 8.13ff., and compare, for example, 12 Arist. Quint. *De Mus.* 5.24ff.). Some attempt has been made to reshape the ideas to fit a Pythagorean framework, and Nicomachus thereby succeeds in introducing a fair measure of confusion. For a masterly transformation of the essential material into a form that Pythagoreans could have used see 11 Ptol. *Harm.* Book 1 ch. 4.

[4] The first part of this sentence paraphrases Aristoxenus (see 7 *El. Harm.* 8.25–9.1, 9.24ff.). The latter part (from 'and which is articulated') presents the idea in Pythagorean terms, in which notes are items with magnitude, placed side by side, not Aristoxenian points on a linear continuum of pitch. The next sentence is also 'Pythagorean', in the same way.

how great is the magnitude possessed by each. Anyone who employs sound otherwise is said not to sing, but to speak. The other kind, the continuous, is that by which we converse with one another or read aloud, when we have no
10 need to make clear the pitches of the notes and to distinguish them from one another, but string out our speech continuously until we have completed what we are saying.[5] Anyone who, while conversing, or recounting something, or reading aloud, makes clear distinctions between the magnitudes associated with each note, dividing and shifting the vocal sound from one to the next, is said not to be speaking or reading, but chanting.[6]

Since then human vocal sound is of two kinds, they believed quite reasonably that there are also two spaces, each occupied by one of them in the course of
20 its motion.[7] The space occupied by the continuous species is by nature unlimited in magnitude, and takes its own particular limit from the point at which the speaker starts to that at which he stops; that is, it is the space between the first utterance and the final silence.[8] Hence the greater part of it is dependent on us. That of the intervallic species, however, is not dependent
240 on us, but is determined by nature, though it too is bounded by distinct operations.[9] Its starting point is the first thing that can be heard, its final point the last which can be sounded.[10] For we begin to be able to grasp and to compare the magnitudes of sounds and the differences between them from the point at which our hearing is first found to operate, though it is possible for there to be produced in nature still fainter sounds, not yet perceptible by us, and for these to escape our notice.[11] In the same way, for instance, there are

[5] This paraphrases the content of 7 Aristox. *El. Harm.* 8.19–25, 9.21–24.

[6] This sentence may refer to the third, 'intermediate' sort of vocal movement introduced at 12 Arist. Quint. *De Mus.* 6.3–5, said there to be used in declaiming poetry. Compare 7 Aristox. *El. Harm.* 9.26–33.

[7] Aristoxenus also speaks frequently of the 'space' (*topos*) in which the voice moves (e.g., 7 *El. Harm.* 14.5, in the passage which is Nicomachus' main source here), but he does not distinguish two different 'spaces'. There is just one, the dimension of pitch, in which both intervallic and continuous vocal movements take place, though in different ways. Nicomachus' conception is not altogether clear. Perhaps he means only that the boundaries of the space are different for the speaking and for the singing voice. Compare 9.2 Theon Smyrn. 52.1ff.

[8] The last phrase echoes 7 Aristox. *El. Harm.* 8.24–5.

[9] The 'nature' that determines this limit cannot be that of intervallic motion abstractly conceived, since it will be said to depend on our powers of hearing and utterance (cf. 7 Aristox. *El. Harm.* 15.2–11). It is then unclear why it is not 'dependent on us', but perhaps this phrase means merely 'not dependent on the individual's whim'. Contrast *El. Harm.* 21.11–17.

[10] This is roughly parallel to Aristoxenus' contention that the smallest usable interval is the smallest that the ear can judge and the voice produce (and these coincide, 7 *El. Harm.* 14.15–24), while the greatest is a little larger for the ear than for the voice (*El. Harm.* 14.25–8). But it turns out that Nicomachus' 'starting point' and 'final point' are not the smallest and largest usable intervals, after all: see notes 11–13 below.

[11] The starting point is then determined by the capacities of our hearing. But Nicomachus now seems to be speaking of the faintness of a sound, rather than its pitch, still less of the size of an interval. Probably there is some confusion here between pitch and volume (cf. 1.19 Archytas frag. 1 with n. 48), though Nicomachus does make the distinction at the beginning of chapter 4. Then his intention may be to speak of the lowest pitched sound that is melodically usable (cf. the opening of chapter 11, and *Exc. ex Nicom.*

bodies which reveal no weight on a balance – bits of chaff or bran or other such things – but when under the addition of a number of such things the beginning of movement is detected, we say that we have the first candidate for the science of weighing. In the same way, when faintness in sound is gradually augmented we make the first thing detectable by the hearing the starting point of the space belonging to the sound proper to song.[12] However, it is not our hearing but the human voice that determines its final limit. The point which it can reach while remaining melodic, and to which it can advance in song, is the one that we identify as the final limit of the space belonging to this kind of vocal sound.[13] But it need make no difference for the present whether we relate this account to the sound we make with our own windpipes, or to that made by instruments, whether stringed, blown or percussive, which are constructed by way of imitation of the sound we make ourselves.[14] Let us pass over the distinction between them for the time being, to avoid fragmenting our exposition right at the outset.

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Chapter 3 That the first music among sensible things is studied in connection with the planets, and that the music that exists among us is studied as an imitation of that one

That the names given to the notes have been taken from the seven stars which move in the heavens and go round about the earth is a view that carries conviction.[15] For they assert that all bodies that rush through a yielding

274.11–20). The 'imperceptible' sounds may not be literally so, but only such that their pitch cannot be assessed by ear. Thus chapter 11 speaks of a 'hoarse whisper' at the bottom of the vocal compass and squeaks at the top, and *Exc. ex Nicom.* of 'croaks and coughs, utterances that are indistinct and unarticulated and unmelodic' as lying beyond the bass range, 'crowings and sounds like the howlings of wolves' beyond the treble. Compare 9.1 Thrasyllus ap. Theon Smyrn. 47.20–48.8, and for a different use of these images 11 Ptol. *Harm.* 10.8–11.

[12] Gradual augmentation of volume brings sounds to the threshold of audibility. The analogy is not obvious in the case of pitch. But pitches are associated with magnitudes, and Nicomachus links greater magnitudes with higher pitches: see especially chapter 10, and the penultimate sentence of chapter 6. Then it is by augmentation in magnitude that a sound is brought up to a pitch that we can assess (compare the analogous but distinct ideas of 8 Eucl. *Sect. Can.* 148–9).

[13] This final limit is evidently the upper boundary of the range of pitch that is melodically usable.

[14] What Nicomachus would say on this issue is not clear, given that some instruments can reach beyond the range of the human voice, nor is it obvious why he insists that the voice and not the ear is the determinant in this case. Possibly the latter thesis is based on the fact that both voices and instruments tail off into 'squeaks' (see n. 11 above) at a point before the ear's capacity to judge pitch gives out. Then the upper limit of melody falls lower in the range than the upper threshold of the ear's judgements. Aristoxenus states explicitly that the ear can judge somewhat larger intervals than the voice can produce (see n. 10 above). On the question whether voice or instruments should provide the criterion, Nicomachus may be relying on an imperfect memory of 7 Aristox. *El. Harm.* 20.28–21.17.

[15] This is of course false: names were assigned to notes quite independently of any astronomical hypothesis, mainly in the context of practice, not of theory (though the origin of some of the names is in doubt).

medium that is readily stirred into waves must make noises differing from one another in their magnitude and in the range of their sound,[16] in relation either to their respective masses or to their particular velocities or to the positions in which the swinging movement of each is completed, such positions being either quite readily disturbed, or the opposite, resistant to being shifted.[17] These same three differences are clearly seen in relation to the planets, which differ from one another in size and in speed and in location,[18] and which rush through the expanse of the aether unceasingly and without rest. That is why each of them has been given the name 'star' [*astēr*], since it is without rest [*staseōs esterēmenos*] and always in swift motion [*aei theōn*]; and for this reason each has been entitled a 'god' [*theos*] and 'aether'.[19]

Now from the course of Saturn, which is highest in relation to us, the deepest note in the octave was named '*hypatē*' ['uppermost'], for what is highest is uppermost.[20] From that of the Moon, which is furthest down of all and circles

[16] 'They' are the Pythagoreans: cf., for example, 1.6 Arist. *De Caelo* 290b. 'Range' translates *topos*, the word rendered as 'space' in the previous chapter. Aristoxenus also uses it both for the 'space' or dimension of pitch in which the voice moves, and for the range within that space that a given note can occupy. The latter seems to be the sense here: see n. 17 below.

[17] The first of these clauses is clear: the pitch ('magnitude') of the note emitted by a planet may depend either on its mass or on its velocity. In translating the latter part of the sentence I follow Levin (1975), pp. 35–9, particularly in her interpretation of *epochai* as 'positions' (for other suggestions see her discussion). The thesis is then that a planet's pitch may depend on its position relative to the other bodies, varying as does the distance between its orbit and those of the others. The irregular apparent movements of the planets encourage the view that the distances between the orbits is not constant. The paths of sun and moon are more stable (compare particularly Arist. *De Caelo* 291b). By assigning stability also to Saturn, Nicomachus is able to associate the three fixed notes of his system with bodies in stable orbits, and the four moveable notes with ones whose orbits shift, sometimes coming closer to an adjacent stable orbit, sometimes moving further away (see the next paragraph). Sun, Moon and Saturn, then, are those whose orbital 'positions' are resistant to being shifted, and the other planets are those whose positions are 'quite readily disturbed'. It will thus be the size of each planet's orbital variation that determines the range, *topos*, through which the pitch of its note can move in changes of genus. Aristoxenus uses the term *topos* in just this way (e.g., 7 *El. Harm.* 22.24ff.), to refer to the range of variation in pitch of such notes as *lichanos* and *parhypatē*. If this account is on the right lines, there are similarities between Nicomachus' theory and that of 11 Ptol. *Harm.* Book III ch. 11; cf. 12 Arist. Quint. *De Mus.* Book III ch. 21 with n. 176.

[18] 'Location' is another rendering of *topos*, which colloquially means merely 'place'. I offer the translation with some hesitation. If it is right, *topos* here is simply equivalent to *epochē*, 'position', in the previous sentence. Alternatively, it still means 'range', in which case the sense is not that the planets differ from one another in the positions of their orbits in space (i.e., in the distances of their orbits from the earth), but that their orbits have different ranges of variation.

[19] These etymological suggestions are entirely fanciful.

[20] Nicomachus now seeks to associate individual bodies with specific notes. Unusually, his system links lower notes with bodies further from the earth. Most authors take the opposite view (e.g., 12 Arist. Quint. *De Mus.* Book III ch. 21, cf. 11 Ptol. *Harm.* Book III ch. 16, though his system is not strictly comparable; Aristotle frag. 203; 2.2 Plato *Rep.* 617b with n. 9; even *Exc. ex Nicom.* 272.9 says that the earliest writers linked the most distant body with the highest note. Boethius *Inst. Mus.* 1.20, 27 follows Nicomachus). For discussions see Levin (1975), pp. 38–42, Burkert (1972), pp. 352–5. Nicomachus' determination to derive the notes' names from their planetary associations may have

the earth most closely, there was taken the name '*neatē*' ['lowest']:[21] for what is furthest down is lowest. From the courses of those which are next to [*para*]
242 each of these there is named on the one hand *parhypatē* (from the one below Saturn, which is that of Jupiter), and on the other *paraneatē* (from the one above the Moon, which is that of Venus).[22] From the one which is in the middle, that of the Sun, which lies fourth in order from each end, there is named *mesē* ['middle'], which is set – at least according to the ancient practice, within the heptachord[23] – at the interval of a fourth from both extremes, just as the Sun also is fourth from each end among the seven planets, and lies in the middle. As to those on each side of the Sun, from the course of Mars, which has been allotted the sphere between the Sun and Jupiter, there is named
10 *hypermesē* ['above *mesē*'], which is also known as *lichanos*;[24] and from that of Mercury, which holds the region between the Sun and Venus, there is named *paramesē* ['beside *mesē*'].[25] We shall fully confirm all these points for you, with

helped to incline him to his heterodox view. Though notions of pitch were rarely expressed, in classical Greek, in terms of 'up' and 'down', this terminology was sometimes used from the fourth century onwards. Hence it could seem odd that a low note was given the name *hypatē*, which in non-musical contexts often means 'highest', as *nētē* means 'lowest'. Nicomachus explains it by linking the lowest note in his system with the planet that is highest in space, and supposing that the note was given its name because of this connection. This is certainly wrong. Probably the notes were named from the positions of the strings or finger-holes used to sound them, *hypatē* being 'furthest away' and *nētē* 'nearest' (again, quite familiar uses of the words). For another conjecture see Sachs (1940), p. 135. Nicomachus may have been influenced by the resonances of the adjectives *hypatos* and *neatos* in religious discourse, where this has astronomical associations too. Thus the Pythagorising Platonist Xenocrates (fourth century B.C.) is said to have distinguished a Zeus *hypatos* (a 'higher' Zeus) who rules in the region of eternal self-sameness, and a Zeus *neatos*, whose province is the world of change below the moon (Plut. *Quaest. Plat.* IX. 1). Xenocrates also spoke of the planets as 'seven gods', and it is possible that Nicomachus drew the ideas of this chapter from a tradition linked to his writings.

[21] *Neatē* is a Doric variant for *nētē*, used probably to give the flavour of antique Pythagoreanism (compare the language of Philolaus, quoted by Nicomachus in chapter 9).

[22] Musically speaking, *parhypatē* is next 'above' *hypatē*, and *paraneatē* (or *paranētē*) next 'below' *nētē*; both are moveable notes. On the relative positions of Venus and Mercury see n. 8 to 2.2 Plato *Rep.* 617a.

[23] Nicomachus reduces the cosmic system to a heptachord by giving no role to the sphere of the fixed stars. His claim for the antiquity of a musical seven-note system falling short of the octave is repeated in chapter 5. The heptachord discussed in the last paragraph of chapter 9 is different. Whether or not he is right (cf. particularly 4.27 ps.-Ar. *Probs* XIX.47), he is plainly motivated at least in part by a wish to assign the invention of the complete octave structure to Pythagoras (in chapter 5).

[24] The usual name for this note has to be treated as a mere variant because *lichanos* means 'forefinger' (the finger used to touch the relevant string, see the table in chapter 11), and quite obviously has no planetary significance. The term *hypermesē* is most unusual: it is formed in the same way as *hyperhypatē* (see nn. 58 and 65 to 8 Eucl. *Sect. Can.*), and the note it names is musically 'below' *mesē*.

[25] This cannot be the note usually called *paramesē*, the upper boundary of the disjunctive tone above *mesē*, since that has no place in the conjunct system of this heptachord. It is true that at the end of chapter 9 Nicomachus describes a bizarre 'ancient heptachord' that spanned an octave, and in which there was a note placed in the usual location of *paramesē*, a whole tone above *mesē*. But he has said that the one intended here treats *mesē* as the common boundary of two fourths, and we are therefore dealing with a

more precision and with diagrammatic and numerical demonstrations, in the treatise which we promised you,[26] noblest of women and best lover of beauty; and we shall explain the reasons why we do not hear this cosmic concord,[27] which utters, as our account sketchily indicates, a sound full to repletion, containing all *harmonia*. But now we must hurry on, for our time goes swiftly, and consider the subjects next in succession.[28]

Chapter 4 That things to do with notes are ordered in accordance with number

20 In general, we say that sound is an impact of air which is unbroken as far as
243 the hearing.[29] A powerful impact or exhalation falling on the surrounding air and striking it in many of its parts results in a large sound, a slight impact makes a small sound, an even one a smooth sound, an uneven one a rough sound; and if it is impelled slowly the sound it makes is deep, if swiftly, the

system spanning a seventh, not an octave (cf. chapter 5). In chapter 7 we are told that each note of the lower tetrachord in this system was at a fourth below its equivalent in the upper, and that the sequence of intervals in the tetrachord, from the bottom upwards, is (in diatonic) semitone, tone, tone. Then the present so-called *paramesē* is a semitone above *mesē*, and is in effect what was usually called *tritē synēmmenōn*.

[26] See the second sentence of chapter 1.

[27] See 1.6 Arist. *De Caelo* 290b with n. 23.

[28] A table of the system sketched in this chapter may be helpful. I have placed the series in astronomical order from highest to lowest, corresponding to a musical order from lowest to highest. The system is equivalent to the pair of tetrachords usually called *mesōn* and *synēmmenōn*. The intervals given presuppose the diatonic genus.

	Saturn	—	*hypatē (mesōn)*	Tetrachord *mesōn*
semitone				
	Jupiter	—	*parhypatē (mesōn)*	
tone				
	Mars	—	*hypermesē (=lichanos mesōn)*	
tone				
	Sun	—	*mesē*	
semitone				
	Mercury	—	*paramesē* (here = *tritē synēmmenōn*)	Tetrachord *synēmmenōn*
tone				
	Venus	—	*paraneatē (=paranētē synēmmenōn)*	
tone				
	Moon	—	*neatē (=nētē synēmmenōn)*	

[29] Cf., for example, 3.15 Arist. *De Anima* 420a, 5 *De Audib.* 800a, and 9.2 Adrastus ap. Theon Smyrn. 50.7, which uses a similar phrase in repeating doctrines attributed to the Pythagoreans. The account has affinities with 1.19 Archytas frag. 1 and the introduction to 8 Eucl. *Sect. Can.* At this point the MSS continue with a passage that is clearly interpolated – the jottings of a reader. 'A note is a breadthless pitch of a melodic sound; and a pitch is a sort of immobility and self-sameness in respect of magnitude of a continuous note. An interval is a sort of route from depth to height or the reverse, and a *systēma* is a combination of more intervals than one.' Cf. nn. 4–6 to 9.1 Thrasyllus ap. Theon Smyrn. 47–8, nn. 12 and 41 to 9.2 Adrastus ap. Theon Smyrn. 49 and 60. Though these sentences can hardly stand where the MSS put them, they may summarise something that was elsewhere in the original text and is now lost, since chapter 12 includes a discussion of these and other conceptions that is said to 'remind' us of things that have already been said.

sound it makes is shrill.[30] Wind instruments, such as *auloi*, *salpinges*, *syringes*, *hydrauloi*, and others like them, are necessarily affected in the opposite way to stringed instruments like the *kithara*, the *lyra*, the *spadix*, and others of that sort. Between these groups, and as it were common to both and affected similarly to both, are found the monochords, which most people call *phandourai* and the Pythagoreans call *kanones*, and also the *trigōnoi* among stringed instruments, as are the *plagiauloi* along with the *phōtinges*, as our account will show later on.[31] In the case of stringed instruments, greater and tauter tensions produce larger and higher sounds, while lesser tensions produce duller and lower ones.[32] For when the plectrum moves the strings, they are displaced from their normal positions, and the tenser ones recoil with a powerful agitation and strike the air about them in many places, as though impelled by the vigorous tension itself, while the slacker ones recoil gently and without agitation, like a carpenter's plumbline. By contrast, in the case of wind instruments the larger cavities and dimensions produce a sluggish and relaxed sound.[33] For if the breath comes out into the surrounding air lacking in tension because of its long journey, and strikes and moves it lethargically, in this way too the sound generated is deep.[34] And it must then be noted that the greater

[30] These remarks can be paralleled in many sources, notably in 5 *De Audib.* Compare 3.17 Arist. *De Gen. An.* 786b ff., particularly for the distinction between a sound's volume and its pitch (see n. 11 above); cf. also the first passage of Adrastus cited in n. 29 above.

[31] For discussions of most of the instruments mentioned here see the index to *GMW* vol. 1. Here I give only a brief indication of their main relevant features. *Auloi* are pipes sounded with a reed, equipped with finger-holes. *Salpinges* are the ancient equivalent of trumpets, though they may occasionally have had a double-reed mouthpiece. The *syrinx*, here as often, is probably the familiar Pan-pipe. The *hydraulos* or *hydraulis* is an organ powered by water-pressure. *Kithara* and *lyra* are the commonest stringed instruments, with strings of equal length. The *spadix* is obscure. Its name means 'branch', specifically 'branch of a palm tree (*phoenix*)'. It is then probably related to the instrument called the *phoenix*, which had a structure like the *lyra* but with palm-branches for its arms (in the list of stringed instruments at Pollux IV. 59, *spadix* appears between *phoenix* and *lyrophoenix*). Quintilian (*Inst. Orat.* 1.10.31) brands the *spadix* as 'effeminate'. That is about the sum of our knowledge of it. The monochord has a single string, stopped at different points to make the notes; normally a tool of theorists, but occasionally used for music making (see 11 Ptol. *Harm.* Book II ch. 12). The *phandoura* or *pandoura* is treated here as identical with the monochord, but sometimes had three strings, and was played rather like a guitar. *Trigōnoi* are triangular harps, with strings graded in length. *Plagiauloi* and *phōtinges* are pipes played transversely, like a modern flute. In some cases they may have had a laterally inserted reed, in others a plain hole, again like a flute.

[32] Nicomachus now begins his explanation of how the three groups of instruments are 'affected' in different ways. The distinctions seem to turn on whether a higher pitch is produced by a greater or a smaller amount of a relevant variable. In the first group discussed (*kithara*, *lyra*, *spadix*) the factor involved is tension, since the strings are equal in length: greater tension goes with higher pitch.

[33] In *auloi*, *syringes* and *hydrauloi* the important variable is length of pipe: greater length goes with lower pitch. But if *salpinges* are trumpets without finger-holes (as they usually were), their different notes are not produced in this way, and it is odd that Nicomachus places them in this group.

[34] Comparable reflections on the causes of pitch-variation in instruments are found in many authors: for an elaborate account see 9.7 Aelianus ap. Porph. *Comm.* 33.16ff. See also 6 Theophrastus ap. Porph. *Comm.* 63.1ff., who like Nicomachus (but for different reasons) points to the effect of several distinct variables, some of which diminish in

and the less come about in relation to quantity, which we ourselves impose by stretching and slackening or by making tubes long or short. Hence it is clear that all these things are ordered by number, since it is of nothing but number that quantity is a proper characteristic.[35]

Chapter 5 That Pythagoras, by adding the eighth string to the seven-stringed lyra, *put together the* harmonia *of an octave*[36]

Pythagoras, first of all men, forming the intention that the middle note should not stand, by conjunction, in the same relation to each of the extremes, and thus display distinctly only the concord of a fourth, in relation both to *hypatē* and to *nētē*,[37] and that we should have a more complex object for study,[38] where the extremes would produce in relation to one another the completest concord, that is the octave, which is in duple ratio, something that cannot be constructed out of two tetrachords, with this aim in view he, first of all men, added an eighth note, fitting it in between *mesē* and *paramesē*, and making it stand at a whole tone from *mesē* and at a semitone from *paramesē*,[39] so that the note which formerly, in the heptachord, was *paramesē*, and which is still third from *nētē*, is called '*tritē*' ('third') and lies in that position no less than before,[40] while that which was inserted is fourth from *nētē*, and forms with it the concord of a fourth, the same concord as that which the original *mesē* made

[35] This is precisely the thesis that Theophrastus (see n. 34) seeks to undermine. Nicomachus has said nothing to explain the 'intermediate' character of his third group of instruments. In the case of *trigōnoi*, monochords, etc., the point seems clear: they are stringed instruments, like the *kithara* and *lyra*, but unlike them they move to a higher pitch by decreasing the relevant variable (length of string). The intention is less plain in the case of *plagiauloi* and *phōtinges*. The idea should be that these, like *auloi*, are wind instruments, but that their pitch is raised by increasing the quantity of a suitable variable. The only likely candidate is pressure ('tension') of breath, which is often a relevant factor in pitching notes on a wind instrument. But these instruments had finger-holes too, and breath-pressure is also influential in the playing of *auloi* (see especially 7 Aristox. *El. Harm.* 41.32–42.14, 43.10–15); it is fundamental in the case of the *salpinx* (cf. 9.3 Adrastus ap. Theon Smyrn. 66.9). Nicomachus has at any rate not said enough to make his claims convincing. Though some further details are offered in chapter 10, he does not make good his promise to explain later his thesis about *plagiauloi* and *phōtinges*.

[36] For the thesis that the Pythagoreans called the octave *harmonia* see chapter 9. Chapters 5–9 are concerned entirely with the structure of the octave. The topics of chapter 4 are revisited in chapter 10.

[37] On the nature of the old structure that Pythagoras is said to have found unsatisfactory see chapter 3. On the shape of this astonishing sentence see n. 1 above.

[38] The claim that the eight-note octave structure was invented for purely intellectual purposes is of course fantastic. It is designed to fit with the attribution of this invention to Pythagoras, whom Nicomachus is inclined to treat as the *fons et origo* of harmonic truth.

[39] The original '*paramesē*' stood a semitone above *mesē* (see n. 25 above). The model that Nicomachus has in mind is that of a new note being inserted, as he says, between the *mesē* and the old *paramesē*, displacing the latter (with the remaining notes in its tetrachord) upwards by a tone. It is thought of as the 'same' note displaced upwards because it is still the third-highest note in the tetrachord to which it belongs (see the next sentence).

[40] On Nicomachus' account, a change of name was essential, since the note was no longer

with *hypatē*. The tone between *mesē* and the newly inserted note (named *paramesē* in place of its predecessor in that position), no matter to which tetrachord it is attached, whether considered as lying higher than the tetrachord involving *hypatē* or as deeper than that involving *nētē*, will display the concord of a fifth, which is the *systēma* made up of the conjunction of the tetrachord itself and the added tone.[41] Now the ratio of the fifth, which is hemiolic [3:2], is found to be a *systēma* of an epitritic [4:3] together with an epogdoic [9:8]; therefore the tone is an epogdoic.[42]

Chapter 6 How the numerical ratios of the notes were discovered

The intervals of a fourth and a fifth, of that which is formed by the combination of the two, known as the octave, and of the tone which lies additionally between the two tetrachords, were established as having this numerical quantity by Pythagoras.[43] The method he adopted was like this. He
246 was plunged one day in thought and intense reasoning, to see if he could devise some instrumental aid for the hearing which would be consistent and not prone to error,[44] in the way that sight is assisted by the compasses, the measuring rod [*kanōn*] and the *dioptra*,[45] and touch by the balance and by the devising of measures; and happening by some heaven-sent chance to walk by a blacksmith's workshop, he heard the hammers beating iron on the anvil and giving out sounds fully concordant in combination with one another, with the exception of one pairing: and he recognised among them the consonance [*synōidia*] of the octave and those of the fifth and the fourth. He noticed that what lay in between the fourth and the fifth[46] was in itself discordant, but was essential in filling out the greater of these intervals. Overjoyed at the way his project had come, with god's help, to fulfilment, he ran into the smithy, and through a great variety of experiments he discovered that what stood in direct relation to the difference in the sound was the weight of the hammers, not the force of the strikers or the shapes of the hammer-heads or the alteration of the

[41] That is, the tone between *mesē* and the new note (now called *paramesē* in accordance with standard usage) is the tone of disjunction between two tetrachords (see particularly 7 Aristox. *El. Harm.* 58.14ff.). A combination of it with either tetrachord will span a fifth.

[42] Cf., for example 8 Eucl. *Sect. Can.* propositions 12 and 13.

[43] The tradition that Pythagoras discovered the fundamental intervallic ratios is very persistent (see, for example, 1.1, 1.2) though others attributed the discovery to his disciples (e.g., 1.3, 1.4). Many of the stories told in this connection (including the present one and 12 Arist. Quint. *De Mus.* Book III ch. 1) are plainly fictions. We cannot be sure even that the discovery was first made by Pythagoreans, though they made notable use of it. It may have been due to practical musicians or instrument-makers, who would have found it useful in the construction of harps (these became popular in Greece during the sixth century B.C., but were widespread in eastern Mediterranean cultures from much earlier times). Most scholars are prepared to give Pythagoras the benefit of the doubt, at least so far as the dissemination of these ideas among the Greeks is concerned.

[44] Compare 11 Ptol. *Harm.* Book I chs. 1–2.

[45] A rod used as an instrument for measuring indirectly the height of tall objects.

[46] That is, their difference, the tone.

iron which was being beaten.[47] He weighed them accurately, and took away for his own use pieces of metal exactly equal in weight to the hammers. Then he fixed a single rod from corner to corner under his roof, so that no variation should arise or even be suspected of arising from the peculiarities of different
247 rods, and hung from it four strings, each of the same material, and consisting of an equal number of strands, and each of equal thickness and twisted to the same extent as each of the others.[48] He then attached a weight to the lower part of each string. And having so contrived it that the length of every string was in all respects absolutely equal, he then plucked strings two at a time in turn, and found the concords previously mentioned, a different concord for each pairing.[49] He perceived that the string under tension from the biggest object attached sounded at an octave in relation to the one under tension from the smallest. The former was of twelve units of weight, the latter of six. Hence he showed that the octave is in duple ratio, as the weights themselves implied. He found that the biggest sounded at a fifth in relation to the smallest but one (which had eight units of weight), and revealed from this that the fifth is in hemiolic ratio, the ratio in which these weights stood to each other. In relation to the one second in weight to itself and greater than the others, which was of nine units, it sounded at the interval of a fourth, in conformity with the relations of the weights. And he at once perceived that this ratio was epitritic, and that this same string was in hemiolic ratio to the smallest (since that is the ratio of 9 to 6): and in the same way the smallest but one, carrying eight units, stood in epitritic ratio to the one that carried six, and in hemiolic ratio to that which carried twelve. And hence he established that what lies between the fourth and the fifth, that is, that by which the fifth exceeds the fourth, is in
248 epogdoic ratio, that in which nine units stand to eight. It was also proved that the octave can be constructed in each of two ways, either in the conjunction of the fifth and the fourth, since duple ratio consists in a conjunction of hemiolic and epitritic – as in the numbers 12, 8, 6 – or the other way round, in the conjunction of the fourth and the fifth, since duple ratio consists in a conjunction of epitritic and hemiolic – as in the numbers 12, 9, 6, which are ordered in that sort of way.[50] Then, having worked on the weights until both hand and hearing were sore, and having established with reference to them the ratios appropriate to their relative positions, he skilfully transferred the

[47] This betrays the legendary character of the tale of this harmonious blacksmith. It is not true that the ratios between the pitches will correspond to those between the weights of the hammer-heads. (See 1.3 with n. 6, 1.4 with n. 11, Burkert (1972), pp. 375ff.)

[48] The matter is hardly so simple. Compare the elaborate procedures described by Ptolemy in 11 *Harm.* Book I ch. 8 and Book III chs. 1–2 for determining whether a string is true along its whole length, and whether two or more have identical properties.

[49] The desired results will again fail to be produced. The pitch-ratios are not directly related to the ratios of the weights, but to those of their square roots. Ptolemy (11 *Harm.* 17.7ff.) knew that the experiment would not work, but his explanation, though it has some truth in it, is beside the point.

[50] The numbers 12, 9, 8, 6 are often used in discussions of this sort, since they are the smallest whole numbers in which this set of ratios can be expressed. An early example is Aristotle frag. 47, quoted in ps.-Plut. *De Mus.* 1139c.

common tying-point of the strings, where they were all suspended together from the diagonal rod, to a stick attached to his instrument, a stick which he called a *chordotonon*, and he transferred the amounts of tension, in the same ratios that were produced by the weights, to a proportionate degree of twist in the *kollaboi* at the upper end.[51] Using this as a foundation and as it were an indubitable indicator, he went on to extend his researches to various kinds of instrument, including beaten pots, *auloi*, *syringes*, monochords, *trigōna* and others like them, and he found the conception arrived at through number to be concordant and immutable in all of them.[52] He named the note characterised by the number 6 *hypatē*, the one characterised by 8 *mesē*, which stands to *hypatē* in epitritic ratio, the one characterised by 9 *paramesē*, which is a tone higher than *mesē* and thus stands to it in epogdoic ratio, and the one characterised by 12 he called *nētē*.[53] And at the same time he filled in the gaps between them according to the diatonic genus, with notes in their proper ratios, and thus made the octachord subservient to concordant numbers, that is, to duple, hemiolic and epitritic ratio, and to the difference between the latter two, the epogdoic ratio.[54]

Chapter 7 Concerning the division of the octave according to the diatonic genus

It was in the following way that he discovered the natural and necessary progression of the scale in the diatonic genus from the lowest note to the highest. (The chromatic and enharmonic genera he later articulated by derivation from the diatonic, as I shall explain to you another time.[55])

[51] On *lyrai* and similar instruments, the *chordotonon* was at the bottom of the sound-box. The *kollaboi* or *kollopes* were devices for adjusting tension (originally strips of raw hide, later pegs or gadgets of various other sorts) fitted to the cross-bar. No Greek musician, of course, had a reliable way of creating the same degree of tension with his *kollaboi* as he had previously produced with weights – except by begging the question and simply adjusting his strings to the same pitches.

[52] Compare, for example, the passages cited in n. 43 above, and 4.19, 4.28 ps.-Ar. *Probs* XIX.23, 50.

[53] Thus Nicomachus (on Pythagoras' behalf) links larger numbers with higher pitches: cf. chapter 11. Some authors, notably Ptolemy, adopt the opposite strategy. For other examples of both approaches see Burkert (1972), p. 380, n. 47 (though I think he is wrong about Archytas). Authors rarely argue for the position they adopt on this issue; an exception is 9.3 Adrastus ap. Theon Smyrn. 65.10ff., whose intriguing argument has some affinities with Nicomachus ch. 10 (but his conclusion is the opposite).

[54] This implies that the tetrachords were filled out with two intervals of a tone each, leaving the ratio of the *leimma* (256:243) for the lowest interval. This is the system of Philolaus, Plato, Euclid, Adrastus, Thrasyllus, and many others in the Pythagorean tradition, but not of the branch that stems from Archytas (see the Appendix to chapter 1).

[55] The 'derivations' of the chromatic and enharmonic that Nicomachus has in mind are comparable to those of 9.5 Thrasyllus ap. Theon Smyrn. 90.22–93.2. They reflect Aristoxenian accounts of a chromatic tetrachord of semitone, semitone, trihemitone, and an enharmonic one of quarter-tone, quarter-tone, ditone (see Nicomachus ch. 12). The more complex analyses recorded in 11 Ptol. *Harm.* Book II ch. 14 cannot be 'derived' from Nicomachus' diatonic in any straightforward way.

However, it appears that it is of the nature of this diatonic genus to involve the following kinds of steps and progressions: semitone, then tone, then tone. That is the *systēma* of the fourth, consisting of two tones and the so-called semitone.[56] Then by the addition of another tone, the one which was inserted in the middle, there is generated the fifth, which is a *systēma* of three tones and a semitone. Following on from this come a semitone, a tone and a tone, making another fourth, that is, another epitritic interval. Hence in the older arrangement, the heptachord,[57] all the notes throughout, from the lowest up, formed the concord of a fourth with the notes fourth in order from themselves, with the semitone changing its position in the tetrachord from first to middle to third in the course of the progression.[58] But in the Pythagorean octachord, whether it is a *systēma* constituted by the conjunction of a tetrachord and a pentachord, or one constituted by the disjunction of two tetrachords separated from one another by a tone, the progression from the lowest string will be of such a kind that all the notes form the concord of a fifth with the notes fifth in order from themselves, with the semitone shifting progressively into the four available positions, first, second, third and fourth.[59]

Chapter 8 Explication of what is said about harmonics in the Timaeus

Now that we have got this far, it will be helpful to explain Plato's opportune remark which he makes during his account of the creation of the soul. He says: 'Thus in each interval there are two means, the one exceeding and exceeded by the same part of its extremes, the other exceeding and exceeded equally in respect of number. And the difference between hemiolic and epitritic intervals is filled up by an epogdoic remainder.'[60]

[56] 'So-called', because it is not an exact semitone (e.g., 8 Eucl. *Sect. Can.* proposition 16); Nicomachus himself makes the point in chapter 12.

[57] See Nicomachus' discussions in chapters 3 and 5.

[58] Nicomachus is probably not thinking of three 'species of the heptachord' analogous to the seven 'species of the octave' generated by the cyclic reordering of intervals (7 Aristox. *El. Harm.* 6.21–31, 36.30–1, 12 Arist. Quint. *De Mus.* 15.9–20). He means only that the interval of a fourth between a note in one tetrachord and its equivalent in the next may take the form semitone, tone, tone; or tone, semitone, tone; or tone, tone, semitone.

[59] The sense is parallel to that explained in the previous note. With the rules of progression mentioned here compare 7 Aristox. *El. Harm.* 29.6ff., 53.33ff., 58.14ff., and 9.2 Adrastus ap. Theon Smyrn. 51.4ff.

[60] This is 2.3 Plato *Tim.* 36a–b, but the second sentence is garbled. Taken as it stands, it reduces Plato's construction of a complete diatonic scale to that of the mere octave framework, fourth – tone – fourth (or fifth plus fourth, where their difference is the 9:8 tone). It also misuses the term *leimma* (here translated 'remainder'), making it refer to the tone. Levin (1975), pp. 87ff. argues that the misquotation is deliberate, designed to suggest that Plato's analysis was less complete than that of the Pythagoreans. She also doubts the authenticity of the material attributed to Philolaus in chapter 9, believing it to be dependent on Plato, whereas Nicomachus wishes to imply that the relation is the other way round, again diminishing Plato's originality. But the Philolaus material differs from Plato's account in important ways. It also involves a complete analysis of a diatonic scale spanning an octave. If Nicomachus meant that Plato's version is based on

A duple interval is that of 12 to 6; and it has two means, the numbers 9 and 8. Now the number 8 is a mean in harmonic proportion between 6 and 12, exceeding 6 by one third of that 6, and exceeded by 12 by one third of that 12. Thus because it is understood to exceed and be exceeded by the extremes by the same part of themselves, Plato called 8 a mean, in the sense that it stands to them in harmonic proportion.[61] For just as the largest bounding number is double the smallest, so the difference between the largest and the middle term, which is 4, is double the difference between the middle term and the smallest, which is 2; and here again the 4 stands to the 2 in duple ratio. A peculiar feature of this sort of mean is that if the extremes are added together and multiplied by the middle term, the result is twice the product of the extremes. Thus eight times the sum of the extremes – that is 18 – makes 144, and that is double the product of the extremes, which is 72.[62]

The other mean, which is 9, and which is so placed as to correspond to *paramesē*, is reckoned to stand as an arithmetical mean in relation to the extremes, exceeding 6 by the same number, 3, as that by which it is exceeded by 12.[63] A peculiar feature of this mean is that the sum of the extremes is twice the middle term, and also that the square of the middle term, in this case 81, is greater than the product of the extremes, in this case 72, by exactly the square of the difference between terms, in this case three times 3, which is 9, since the difference in this example is 3.[64] One can also display a third mean, in what is known as ruling proportion, consisting in both these middle terms, 9 and 8. For 12 stands to 8 as does 9 to 6, the ratio being hemiolic in both cases. And the product of the extremes is equal to that of the middle terms, twelve times 6 being equal to nine times 8.[65]

his, he could hardly deny that Plato knew of the whole structure. Again, Nicomachus says later (chapter 11) that he will set out elsewhere a complete 'division of the *kanōn*' according to the system of Timaeus of Locri, 'whom Plato also followed'. This can only mean that Plato's analysis in the *Timaeus* (whether plagiarised or not) located every note of the scale. Levin's diagnosis, I think, must fail (I would also accept the authenticity of the Philolaus passage). So, far from seeking to diminish Plato, Nicomachus like many others is enrolling him as an honorary Pythagorean. But I do not know why the *Timaeus* quotation appears here in this truncated and distorted form.

[61] This represents accurately Plato's conception of this mean, which he almost certainly derived from Archytas.

[62] These additional calculations are correct, and help to give the sense of mathematical coordination. Other authors add different characterisations of this mean, e.g., Theon Smyrn. 114.14ff. Like Theon, Nicomachus discusses the forms of mean and proportion mentioned here, and a series of others, in his *Intr. Arithm.* Book II chs. 27–9. On harmonic proportion see 27.5. On Plato's three forms of proportion see also Proclus *In Timaeum* III, 171.20–174.10 (Diehl).

[63] This again represents Plato's intentions accurately.

[64] The first point in this sentence is also made, for example, at Theon Smyrn. 113.22–5. A string of properties of terms in arithmetic proportion is listed at Nicomachus *Intr. Arithm.* Book II. 27.3.

[65] 'Ruling' proportion is discussed at length in Nicomachus *Intr. Arithm.* Book II ch. 29, where it is called 'the most perfect' proportion. Iamblichus calls it 'musical'.

Chapter 9 Evidence for what we have said, derived from Philolaus

Even the most ancient writers show agreement with what we have explained. Their name for the octave is '*harmonia*', for the fourth '*syllaba*' (since it is the first concordant combination [*syllēpsis*] of notes), and for the fifth '*di' oxeian*' (since the fifth is continuous with the concord first generated and goes on upwards);[66] and the combination of both *syllaba* and *di' oxeian* together is the *dia pasōn*, and was given the name '*harmonia*' because it is the first concord to be fitted together out of concords.[67] Their agreement with what we have said is made clear by Philolaus, the disciple of Pythagoras,[68] who writes roughly as follows in the first book of his *Physics*: pressure of time demands that we rest content with just one witness, though many people say similar things in various ways about the same subject. Philolaus' statement goes like this.

'The magnitude of *harmonia* is *syllaba* and *di' oxeian*. The *di' oxeian* is greater than the *syllaba* in epogdoic ratio. From *hypatē* to *mesē* is a *syllaba*, from *mesē* to *neatē* is a *di' oxeian*, from *neatē* to *tritē* is a *syllaba*, and from *tritē* to *hypatē* is a *di' oxeian*. The interval between *tritē* and *mesē* is epogdoic, the *syllaba* is epitritic, the *di' oxeian* hemiolic, and the *dia pasōn* is duple. Thus *harmonia* consists of five epogdoics and two dieses; *di' oxeian* is three epogdoics and a diesis, and *syllaba* is two epogdoics and a diesis.'[69]

It must be remembered that what he here calls '*tritē*' is the *paramesē* of the heptachord which there was before the additional insertion of the disjunctive tone in the octachord.[70] For this note used to stand at an undivided interval of three semitones from *paraneatē*,[71] from which interval the string inserted next to it cut off a tone, and there was left a semitone as the remainder between *tritē* and *paramesē* in the disjunction.[72] It is then correct to say that the old *tritē*

[66] On these names see 12 Arist. Quint. *De Mus.* 15.8–10 with n. 99.

[67] *Dia pasōn*, 'through all (the strings)' is the usual expression for 'octave'. This explanation of the old use of *harmonia* derives from the fact that it means a 'fitting together', not necessarily a musical one.

[68] Levin (1975), p. 85 takes this expression to imply that Philolaus was a direct pupil of Pythagoras (so emphasising Philolaus' antiquity and authority). In that case Nicomachus would be quite wrong (Philolaus lived in the late fifth century). But it need only mean that he was a 'follower' of Pythagoras' doctrines.

[69] For commentary on this passage see notes to 1.12 Philolaus frag. 6.

[70] This seems to suggest that Philolaus' system is the 'old heptachord' of chapters 3 and 5. But that was a pair of conjoined tetrachords spanning a seventh, in which the so-called *paramesē* stood a semitone above *mesē* (in the place of the note usually called *tritē synēmmenōn*). In Philolaus' system, the *tritē* is a tone above *mesē*, at the site of the *paramesē* of the normal octachord that was formed by two tetrachords disjoined by a tone. This system spans an octave.

[71] Thus the upper tetrachord, beginning at the so-called *tritē*, a tone above *mesē*, was defective, containing only three notes: *tritē* was three semitones below *paraneatē*, and *paraneatē* a tone below *neatē*. Hence, the octave was spanned by an arrangement of only seven strings: see 1.12 with n. 34.

[72] The new, post-Philolaan note is treated as being inserted below the old '*tritē*', and as displacing it upwards by a semitone. The diagnosis is parallel to that given for a similar case in chapter 5 (see n. 39 above). The *tritē* then comes to stand where diatonic *tritē diezeugmenōn* normally does, a tone below *paranētē*, and *paramesē* takes its place a tone above *mesē*.

stood at an interval of a fourth from *nētē*, the interval that *paramesē* now marks off, instead of *tritē*. But people who do not understand this make the criticism that it is impossible for *tritē* to be in epitritic ratio with *nētē*. And others quite plausibly say that the additional note was inserted not between *mesē* and *tritē*, but between *tritē* and *paraneatē*: and they say that this note was called *tritē* in place of the other, and that the old *tritē* becomes *paramesē* in the disjunction.[73] And they say that Philolaus calls *paramesē* by its former name,
254 *tritē*, even though it stands at a fourth from *nētē*.

Chapter 10 Concerning the fitting together [harmosis] of the notes through the numerical ratios

Let us go back to our earlier discussion[74] and add to it the next step, which is to assert that by contrast with the proportionality [*symmetria*] taken in relation to tension, which makes the notes high through large quantity and low through
10 small, when we consider the lengths or thicknesses of strings, or of the bores of *auloi*, we do so according to the reverse proportion [*analogia*]. For in these cases it is the other way round, small dimensions yielding high pitch and greater ones low pitch. For if you take a long string at a single uniform tension lying above a measuring rod [*kanōn*], and fixed away from the rod so as not to touch it, and if you compare the note from the whole string when it is plucked with that from half of it, the string being divided off exactly in the middle by a bridge or something of the sort, so that the vibration from the blow does not extend further than half-way, you will find that the sound from the
20 half string stands at an octave to the larger sound from the whole – that is, it is double that sound, being qualified in the opposite way to the relationships of the lengths.[75] And if you exclude the vibration from a third part of the length, accurately measured, the sound uttered by two thirds of the string will necessarily stand in hemiolic ratio to that from the whole, the opposite way round to the length. And if you cut off a fourth part of the string from the blow,
255 not allowing the vibration to extend further, the sound from three parts of the string will stand in epitritic ratio to that from the whole, the opposite way round to the relationship of length. In the same way in the case of an *aulos*

[73] This way of viewing the matter may seem more natural. A new note is inserted a semitone above the old *tritē*, and because it is now the third note from *nētē* (inclusive), it takes over the name *tritē* ('third'). The old *tritē* stays where it was, and changes its name to *paramesē*.

[74] Chapter 5–9 are the core of the treatise, and have been presented as a historical narrative. This chapter returns to the analytic, non-historical mode, and its subject is continuous with that of chapter 4.

[75] This explains why monochords are treated in chapter 4 as having affinities with *auloi* etc. The notion of 'reversing the relationships' is made much of in Thrasyllus' account of the division of the *kanōn*, 9.4 Theon Smyrn. 87.9ff. Nicomachus' phrase 'double that sound' reveals that he does not regard the two kinds of relation as equally valid expressions of the sounds' pitch-ratios: greater numbers belong properly to higher notes (see n. 53 above). Nicomachus probably rests his view on the familiar association of higher pitch with greater speed: for a detailed discussion see 9.7 Aelianus ap. Porph. *Comm.* 36.9ff., and cf. 11 Ptol. *Harm.* Book 1 ch. 3.

divided into four equal lengths by three holes, if the holes are sealed up by covering them with the fingers, and we compare the note from the whole *aulos* with that given out from the middle hole when the finger covering it is lifted,
10 the latter will be found to be double the former, and the sound from the middle hole will be at an octave from that of the whole *aulos*. And this middle note stands in hemiolic ratio to the note from the hole below it, which lies lowest down, next to the bottom of the instrument; while this one is in epitritic ratio to the note from the whole. The sound from the hole nearest the mouthpiece [*glōssis*] is double that from the middle hole, and quadruple that from the whole, the reverse of the proportions of the lengths. And in *syringes* the lengths produce a similar result, and the widths of the pipes function in the same way
20 as the thicknesses of strings: for the sound of a pipe in two sections is double that of one in four.[76]

Chapter 11 Concerning the double octave in the diatonic genus

Now the scope of the diagram in the diatonic genus is a double octave of quadruple breadth.[77] And this is the maximum that a voice trained for
256 competition can traverse without some kind of risk or slippage, since it becomes difficult to pitch at each of the extremes, tending towards a mere squeak in the area of *nētē* and to a hoarse whisper around the deeper of the *hypatai*.[78]

To the ancient style of *lyra*, that is the seven-stringed one, put together by the conjunction of two tetrachords (in which *mesē* formed a boundary for both the concordant intervals, the upper boundary for the lower tetrachord, which goes down to *hypatē*, and the lower boundary for the upper tetrachord, which
10 goes up to *nētē*),[79] they added two more tetrachords, one at each end. Next to the original *nētē* they added what is called the tetrachord *hyperbolaiōn*, since it consists of higher and additional sound, taking it in conjunction, and beginning from the old *nētē* itself. Then the added tetrachord reached its limit by the attachment of only three further notes, which were given, reasonably

[76] Compare particularly 9.2 Theon Smyrn. 61.2ff., and, for example, 4.19 ps.-Ar. *Probs* XIX.23. Nicomachus' statement about the widths of pipes is misleading. Pitch does not vary with pipe-width or string-thickness in the same way as it does with pipe-length or string-length, nor do pipe-width and string-thickness affect pitch in equivalent ways, but compare 9.2 Theon Smyrn. 57.5. The reference to 'sections' suggests that the pipes of the *syrinx* were sometimes made up of different numbers of pieces, each of the same size, but the inference is insecure. The wing-shaped *syrinx* common in the Roman period seems usually to have had continuous, not 'modular' pipes. In the older Greek form the pipes were all the same length, blocked to different sounding lengths with wax (see *GMW* vol. 1, p. 196, n. 46).

[77] That is, the ratio between notes bounding an interval whose breadth is a double octave is 4:1.

[78] See chapter 2 above with n. 11. The deeper of the *hypatai* is *hypatē hypatōn*. For a rather larger estimate of the compass of the human voice see 7 Aristox. *El. Harm.* 20.28–31. Compare also 9.3 Adrastus ap. Theon Smyrn. 64.1ff.

[79] This describes the heptachord of chapters 3 and 5, not that attributed to Philolaus in chapter 9.

enough, the following names: *tritē hyperbolaiōn*, then *paranētē hyperbolaiōn*, then *nētē* of the same tetrachord. In order to make the distinction, the notes of
20 the previous tetrachord – the one conjoined with *mesē* – should be given the following names: after *mesē*, *tritē synēmmenōn*, then *paranētē synēmmenōn*, then *nētē synēmmenōn*. And when the whole construction upwards from *mesē*
257 is enumerated, it too necessarily completes a heptachord. Going down from the original *hypatē* they added the second of the tetrachords mentioned, again in conjunction, with the old *hypatē* being incorporated in it as well, as the highest of the notes in it. In the same way as before, in order to distinguish this from the previous arrangement, it also was given clearer names. To each was added the term *hypatōn*, giving names like *hypatē hypatōn*, *parhypatē hypatōn*, and
10 *diatonos hypatōn* or *lichanos hypatōn*, for it makes no difference which of these expressions is used. And this whole *systēma* from *mesē* to *hypatē hypatōn* also came out as a heptachord composed of two conjoined tetrachords, employing as their one common note the old *hypatē*. Thus from *hypatē hypatōn* to *nētē hyperbolaiōn* there were four conjunct tetrachords. And so a thirteen-string *systēma* was discovered, with seven strings arranged diatonically in both directions.[80] Later, as we said before, those who elaborated the *harmonia* inserted the eighth note, the one which stands at the interval of a
20 tone, between *mesē* and the old *tritē* (or, as some say, between *tritē* and *paranētē*), thus displaying the character of the fifth.[81] And *mesē* was then found
258 to be no longer really *mesē*. For where there is an even number of strings, there can be no single middle one, but there must be two, the seventh and the eighth. Further, they took in addition below *hypatē* one outermost note, the lowest of all – which for that reason they called *proslambanomenos* – this note too being at a distance of a tone, below *hypatē hypatōn*, so that there should be octachord *systēmata* in both directions from *mesē*, while *mesē* itself became genuinely the middle note, since it lay eighth from each end in a series of fifteen
10 notes, and so that the complete magnitude of the diagram should become two octaves, that is, the double taken twice, which is quadruple.[82] The arrangement of the names, taken in sequence upwards, was thus as follows.

[80] Nicomachus' description is admirably clear, but the thirteen-note system that he describes is otherwise unknown (it reappears in the thirteenth-century treatise of Pachymeres, *Harm.* ch. 11, 127.10–12 (Tannery)). Its oddity is the addition of a fourth tetrachord in conjunction above *nētē synēmmenōn*, forming a complete system of four conjoined tetrachords spanning altogether two sevenths, not two octaves. The tetrachord *hyperbolaiōn* is thus placed a tone lower than usual. It was in any case not standard practice to conceive this tetrachord as continuing a scale from the tetrachord *synēmmenōn* at all, even through a tone of disjunction (see particularly Cleonides *Eisagoge* 199.11–201.7), but this disjunction is recognised at Bacchius *Eisagoge* 301.10–16, and see chapter 11 below. For a discussion of Nicomachus' scale see Chailley (1956), but the system may be pure invention.

[81] This refers to the 'invention' made by Pythagoras, chapter 5 above.

[82] The result is the perfect system spanning two octaves, as described by many authors. Nicomachus has not explained that the insertion of the new note (above *mesē*, or above the old *tritē*) has transposed the tetrachord above *mesē* upwards by a tone to form the tetrachord *diezeugmenōn*, and that this change has also displaced the tetrachord *hyperbolaiōn* upwards through a tone. *Nētē hyperbolaiōn* is now in its usual position, an octave above *mesē*. The name *proslambanomenos* means 'taken in addition'.

(1) *Proslambanomenos*

(2) then, after a distance of a whole tone, *hypatē hypatōn*

(3) then after a semitone, *parhypatē hypatōn*

(4) then after a tone, *lichanos hypatōn*, so named because the finger of the left hand next to the thumb, which is called by this name, '*lichanos*', is always placed on this string[83]

(5) then after another tone, *hypatē mesōn*

20 (6) next in succession [*hexēs*], after a semitone, *parhypatē mesōn*

(7) after a tone, *lichanos mesōn*, which they also call '*diatonos*' after the diatonic genus itself

(8) then after another tone, *mesē*

(9) then *paramesos*, after a whole tone[84]

(10) then *tritē diezeugmenōn*, after a semitone

259 (11) then after a tone, *paranētē diezeugmenōn*

(12) and after another, *nētē diezeugmenōn*

(13) next in succession [*hexēs*] to this, after a semitone, *tritē hyperbolaiōn*

(14) then after a tone, *paranētē hyperbolaiōn*

(15) and above them all, after a tone, *nētē hyperbolaiōn*

In a way that is reminiscent of the conjunction involved in the original heptachord, there was also inserted, between the tetrachord *mesōn* and the tetrachord *diezeugmenōn*, another, called the tetrachord *synēmmenōn*, which
10 begins with its own *tritē* a semitone away from *mesē*, then, after a tone, has a *paranētē* peculiar to itself, and then, after another tone, has the *nētē synēmmenē*, which is in all respects of the same tension and sound as *paranētē diezeugmenē*.[85] Thus there are in all five tetrachords, the tetrachords *hypatōn*, *mesōn*, *synēmmenōn*, *diezeugmenōn* and *hyperbolaiōn*: and these involve two disjunctions and three conjunctions. The disjunctions are those between the tetrachords *synēmmenōn* and *hyperbolaiōn*,[86] and between *mesōn* and *diezeugmenōn*, each disjunction making a separation of the magnitude of a
260 tone: and the three conjunctions are the one conjoining *hypatōn* with *mesōn*, the one conjoining *mesōn* again with *synēmmenōn*, and lastly the one conjoining *diezeugmenōn* with *hyperbolaiōn*. In our broader treatment of the subject we shall set out for you, for each note of these tetrachords, the ways they were discovered and their causes and their progressions, how these discoveries came about, and by whom, and when, and on what basis, beginning from the tetrachord and going on to the most complete filling out of the octave,

[83] Compare chapter 3 above and n. 24; the same explanation is given, for example, at 12 Arist. Quint. *De Mus.* 8.17–19.

[84] The usage *paramesē* is more common, but see, for example, 12 Arist. Quint. *De Mus.* 7.24, 8.7.

[85] More commonly *synēmmenōn*, *diezeugmenōn*.

[86] See n. 80 above.

not only in this diatonic genus, but also in the chromatic and enharmonic, using the ancient evidence of the most trustworthy and eminent men. In addition we shall set out the division of the so-called Pythagorean *kanōn*, worked out accurately and completely according to the doctrine of that school, not in the manner of Eratosthenes or Thrasyllus, who misunderstood it,[87] but in that of Timaeus of Locri, whom Plato also followed, right up to the twenty-seven-fold ratio.[88]

Chapter 12 Concerning the progression and division of notes according to the three genera[89]

So that you may have in good order the progressions according to the three genera, extending from *proslambanomenos* to *nētē hyperbolaiōn*, it will be useful for the sake of clarity to begin by reminding you first of a few points which have already been mentioned above.

A note [*phthongos*] is an indivisible sound, as it were a unit in respect of hearing: or, according to more recent writers, it is the incidence of sound on a single simple pitch: or, as some say, it is a sound with no breadth, having no extension in space.[90]

An interval [*diastēma*] is what lies between two notes.[91] A relation [*schesis*] is the ratio which measures the distance in any interval: and a difference [*diaphora*] is the excess or deficiency of one note with respect to another. Those

[87] The complaint against Thrasyllus may be that he took his divisions only through the compass of the musical octave or double octave, unlike Plato, whose cosmic scale runs, as Nicomachus says, up to the ratio 27:1 (four octaves and a sixth). For a defence of this aspect of Plato's project see 9.3 Adrastus ap. Theon Smyrn. 64.1–65.9. Alternatively, or additionally, Nicomachus may be recommending the principle that division should be accomplished on the basis of a scheme of proportions (see chapter 8 above), rather than by the mere application of ratios, even if the methods produce the same result. Possibly he takes a critical view of some of the details of Thrasyllus' procedure, for example, those in which it differs from that of 8 Eucl. *Sect. Can.* propositions 19–20. For Thrasyllus' divisions see 9.4–9.5 Theon Smyrn. 87.9ff. Those of Eratosthenes are tabulated in 11 Ptol. *Harm.* Book II ch. 14, and are certainly open to other objections from Nicomachus' point of view: see n. 117. Nicomachus would find fault with the actual ratios of his divisions, unlike those of Thrasyllus, since they are not direct derivatives from the Platonic system, and his general procedure is arguably incoherent.

[88] The character in Plato's dialogue who sets out the division of the world-soul is of course Timaeus. There were insinuations even in the fourth century (retailed by Aristoxenus) that the dialogue was plagiarised from Pythagorean sources. But Nicomachus may be alluding to a treatise that passed in later antiquity for the work of Timaeus of Locri. It is in fact a pseudo-Pythagorean forgery, whose date is not much earlier than that of Nicomachus himself, loosely based on Plato's *Timaeus*. In its scalar constructions, however, it does not even use the number 27. (For comments and references see Levin (1975), pp. 91–2. The spurious treatise is printed in Thesleff (1965), pp. 205–25.)

[89] This will fulfil the promise made at the opening of chapter 7: see n. 55 above.

[90] For a definition of the first sort see 12 Arist. Quint. *De Mus.* 7.15–16. The second is derived ultimately from 7 Aristox. *El. Harm.* 15.15–16, the third from *El. Harm.* 3.21–4. For other references see n. 37 to Arist. Quint. *De Mus.* 7.15–16. This 'reminder' and those that follow echo nothing earlier in the text, but see n. 29 above.

[91] Though the immediate sequel reintroduces Pythagorean conceptions, this definition, like most of the chapter, belongs to the Aristoxenian tradition. See 7 *El. Harm.* 15.24–32 and, for example, 12 Arist. Quint. *De Mus.* 10.18–19.

who think that relation and difference are the same are wrong: for as you will understand, the difference between 2 and 1 is the same as that between 1 and 2, whereas their relation is not the same. For 2 is double 1, while 1 is half of 2. Or again, the difference between three or more terms in arithmetic proportion is the same in each case, but the relation is different in each. But you will learn more fully about this in our fuller treatment.[92]

A *systēma* is an assemblage of two or more intervals.[93] Now while no note in these intervals is concordant with its successor, but always discordant, some *systēmata* are concordant and others discordant. They are concordant when the notes which bound them are different in magnitude, but when struck or sounded simultaneously, mingle with one another in such a way that the sound they produce is single in form, and becomes as it were one sound. They are discordant when the sound from the two of them is heard as divided and unblended.[94]

Since the first and most elementary concord is the fourth in a continuous tetrachord and in epitritic ratio, it is naturally here that the differences between the three genera of melody are to be found.[95] The diatonic, about which we spoke earlier, proceeds as follows: a semitone, then a tone, then a tone, three intervals between four numbers, that is, four notes. And this is why it is called 'diatonic', because it is the only one of them to proceed through tones [*dia tōn tonōn*].[96] The chromatic progresses like this: a semitone, then another semitone, then above them an undivided interval of three semitones, so that it too, even though it is not straightforwardly composed of two tones and a semitone, nevertheless evidently has intervals equal to two tones and a semitone. It is in the nature of the enharmonic to have the following division: a diesis – that is, half of a semitone – and then another diesis, together equal to a semitone, and then the remainder of the tetrachord, a whole, incomposite ditone. Thus this is also equal to two tones and semitone: for no note can be concordant with another inside these limits. So it is clear that the distinctions

[92] This simple distinction between 'difference' and 'relation' is not the same as that made by some authors between 'interval' (*diastēma*) and 'ratio' (*logos*). See, for example, Theon Smyrn. 81.6–82.5, where the essential points are that the interval between A and B is the same as that between B and A, while the ratios A:B and B:A are different, and that there can be a ratio, but no interval, between two items that are equal. The former claim sounds like the first of those made by Nicomachus, but in Theon it is clear that 'interval' is not equivalent to 'arithmetical difference', while this is plainly the sense of 'difference' in Nicomachus' point about arithmetical proportion.

[93] This is equivalent to 7 Aristox. *El. Harm.* 15.34–16.1. For other references and alternative descriptions see 12 Arist. Quint. *De Mus.* 13.4–5 with n. 81.

[94] On these conceptions of concord and discord see 8 Eucl. *Sect. Can.* 149.17–20 with n. 7. For the notion of *systēmata* as concordant or discordant see 7 Aristox. *El. Harm.* 17.5–7, 12 Arist. Quint. *De Mus.* 14.7–10.

[95] Compare 7 Aristox. *El. Harm.* 21.34ff., 46.19ff. The analyses that follow are based entirely on Aristoxenus, corresponding to his tense diatonic, his tonic chromatic and his enharmonic: cf. 9.2 Adrastus ap. Theon Smyrn. 53.17ff. It is only after the analyses have been set out that Nicomachus alludes to the Pythagorean view that divisions into half-tones and quarter-tones are impossible.

[96] Compare 9.2 Adrastus ap. Theon Smyrn. 54.12–15, and the interpolated passage at 12 Arist. Quint. *De Mus.* 92.22–3.

between the genera do not arise from the four notes of the tetrachord, but only from the two central ones.[97] Thus in the chromatic the third note is different from that in the diatonic, while the second remains the same as that of the diatonic while being of the same pitch as the third note of the enharmonic. In the enharmonic the two middle notes are different from those of the diatonic; and thus the enharmonic lies at the opposite pole to the diatonic, and the chromatic is between them. For it involves only a slight alteration, by just one semitone, from the diatonic. (Hence we say of changeable people that they have 'colour' [*chrōma*].)[98] The extremes of the tetrachord are called fixed notes, since they do not change in any of the genera. The notes in the middle are moveable – or at any rate they are in the enharmonic. In the chromatic the second note is both moveable and not moveable: it does not change with respect to diatonic, but with respect to enharmonic it does.

The octave, which is the *systēma* in eight strings from *mesē* down to *proslambanomenos*, or from *mesē* up to *nētē hyperbolaia*, granted that the fourth is two tones and a semitone, and the fifth three tones and a semitone, does not straightforwardly add up to six tones, as more recent writers think, but to five tones and two so-called semitones. If these were really halves of tones, what would prevent them making up a tone together, and the octave being six tones?[99] But we shall give a clear and detailed demonstration of this point in our fuller account. Philolaus is in agreement with us, since in the work previously mentioned he says: '*Harmonia* is five epogdoics and two dieses', that is, two semitones: and these together would have made one tone, if they were really half a tone each.[100]

When the three genera are mixed together with one another in the same diagram, their names will be as follows.[101]

Proslambanomenos	A
Hypatē hypatōn	B
Enharmonic *parhypatē hypatōn*	B+
Chromatic and diatonic *parhypatē hypatōn*	c
Enharmonios hypatōn	c

[97] See the references to Aristoxenus in n. 95 above, and for a formal argument 7 *El. Harm.* 61.5ff.

[98] Compare 9.2 Adrastus ap. Theon Smyrn. 55.4–7, 12 Arist. Quint. *De Mus.* 92.24–5.

[99] Nicomachus does nothing to prove that these addition sums are wrong, and his expression is a little confused. But the general point is clear enough: the relevant doctrines are those set out in 8 Eucl. *Sect. Can.*, particularly propositions 3, 9, 14–16. Compare also 9.2 Adrastus ap. Theon Smyrn. 53.8–16, 9.9 Panaetius ap. Porph. *Comm.* 65.21ff.

[100] See the Philolaus passage quoted in chapter 9.

[101] The table is unusual only in listing enharmonic *parhypatai* and *tritai* separately from those of diatonic and chromatic. Most writers do not treat them as distinct entities for these purposes. The fact may ultimately be traceable to the early existence of systems in which the second-lowest note of the tetrachord was not moveable (see 1.21 Archytas ap. Ptol. *Harm.* 30.9ff.). The names *enharmonios*, *chrōmatikē*, *diatonos* are common variants for 'enharmonic (etc.) *lichanos*' and 'enharmonic (etc.) *paranētē*'.

Chrōmatikē hypatōn	c♯
Diatonos hypatōn	d
Hypatē mesōn	e
Enharmonic *parhypatē mesōn*	e+
Chromatic and diatonic *parhypatē mesōn*	f
Enharmonios mesōn	f
Chrōmatikē mesōn	f♯
Diatonos mesōn	g
Mesē	a
Enharmonic *tritē synēmmenōn*	a+
Chromatic and diatonic *tritē synēmmenōn*	b♭
Enharmonios synēmmenōn	b♭
Chrōmatikē synēmmenōn	b
Diatonos synēmmenōn	c′
Nētē synēmmenōn	d′
Paramesē	b
Enharmonic *tritē diezeugmenōn*	b+
Chromatic and diatonic *tritē diezeugmenōn*	c′
Enharmonios diezeugmenōn	c′
Chrōmatikē diezeugmenōn	c′♯
Diatonos diezeugmenōn	d′
Nētē diezeugmenōn	e′
Enharmonic *tritē hyperbolaiōn*	e′+
Chromatic and diatonic *tritē hyperbolaiōn*	f′
Enharmonios hyperbolaiōn	f′
Chrōmatikē hyperbolaiōn	f′♯
Diatonos hyperbolaiōn	g′
Nētē hyperbolaiōn	a′

Forgive the hasty nature of this essay: as you are aware, you set me this task just as I was poised to go off on my journey: and with your accustomed great kindness and thoughtfulness for your friends,[102] accept it as a beginning and a friendly offering. You can expect, if the gods are favourable, a most thorough and altogether a most complete technical treatise on these matters, which I will send you just as soon as the first opportunity arises.

[102] The phrase is difficult, and might possibly mean 'intellectual concern about common matters' (i.e., about matters of general or abstract interest). This conclusion picks up the remarks of the first chapter, and rounds the work off as a letter to the lady it addresses.